

# Online Integrity Alert Limit Determination Method for Autonomous Vehicle Navigation

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**Abstract.** Integrity is the critical performance indicator for navigation in safety-critical applications such as autonomous vehicles. Alert limit is one of the representative parameter in integrity which defines the maximum tolerable positioning error for an operation to safely proceed. However the integrity requirements for GNSS assessment are quite different from those for autonomous vehicles. For autonomous vehicles, a reasonable alert limit needs to ensure the vehicle security and take full advantage of the space between vehicle and lane as much as possible. Based on the analysis of differences from civil aviation to autonomous vehicles, an improved alert limit determination method is proposed in this paper. The kinematic model is firstly introduced into the online determination of alert limit. The integrity risk on two sides are allocated optimally respect to the road geometry and kinematic model. The fixed cuboid bounding box is replaced by a subversive fan-shaped bounding box which is more reasonable to cover the safe-critical areas. The experiment test results compared with those of the Ford model also verified the superiority of the proposed method. Finally the paper also gives the alert limits calculated based on the Chinese standards and hopefully it could provide some references.

**Keywords:** Integrity · Autonomous Vehicles · Alert limit · Kinematic model.

## 1. Introduction

Autonomous vehicles are the next technology revolution in transportation and will greatly improve the safety, efficiency and intelligence. Autonomous vehicles require an extremely accurate, robust, and reliable navigation system to guarantee the mission accomplishment and operation safety [ 1 ]. The complexity and

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diversity of urban environment further aggravate the requirements for localization and navigation [2].

As autonomous vehicles require decimeter-level even centimeter-level positioning accuracy, most of the current researches focus on the robust and reliable navigation solution based on multi-sensors [3 - 7]. It results that the performance assessment system is got less attentions in above safety-critical application. To some extent, the navigation requirements are much more important as they determine the status of safe and define the performance of sensor solutions at scale. Besides the accuracy, integrity is another representative indicator among the positioning, navigation and timing (PNT) applications [8]. Different from the traditional fault detection technology, integrity puts more emphasis on the measure of trust that can be placed in the correct position and the ability to provide timely alert when the navigation system should not be used for navigation [9]. Integrity was firstly introduced in GPS and accepted by the civil aviation as one of the crucial criteria for satellite navigation system [10]. The corresponding concepts such as probability of hazardous misleading information (PHMI), alert limit (AL) and protection level (PL) are defined and used for integrity evaluation [11,12]. Actually as a representative quantifiable criterion, integrity has been introduced and researched in many fields.

On the basis of successful and mature application in civil aviation, the definition of integrity risk and bounding box are firstly introduced by Ford Motor Company (referred to 'Ford model' hereinafter) into autonomous vehicles to evaluate the accuracy and integrity [13]. Considering the vehicle dimensions and road diversity, a baseline algorithm to calculate the alert limits is given in details in lateral and longitudinal components. The accuracy is also given based on the relationship with integrity in Gaussian distribution. The Ford model made full use of bounding box in global navigation satellite system (GNSS) position of civil aviation. A fixed cuboid box is defined for allowed position error and the corresponding protection levels are then determined. The mode needs to find a trade-off between the lateral and longitudinal alert limits. It is easy to understand that the balance needs to fall to the lateral component as it is more stringent. However, different from the civil aircraft, whose trajectory is smooth and the route in flight is relatively vast, the challenge that autonomous vehicle facing is the complexity and limitation of lanes. In most cases, the width of lane is less than 4 meters. With the road curvature increasing, the size of the cuboid box the lane can contain is drastically decreased. Actually the size of box is severely limited in curved road, resulting in a more restrict and conservative alert limits in final vehicle operation. But for integrity, conservative alert limits will affect the availability of navigation system. What's worse, the model results show that the vehicle have to drive off the centerline in curved road and close to the inner side to guarantee the biggest box. The added complexity and uncertainty to the control and navigation system make the loss outweighs the gain.

As an expanded but realistic alert limit is great of help to improve the availability of navigation system and autonomous vehicle operation in complex

environments, an improved alert limit determination method is proposed in this paper. Enhanced by the vehicle kinematic model, the traditional cuboid bounding box is replaced by a subversive fan-shaped bounding box which can take full advantage of the space between vehicle and the lane. The experiment results show that the proposed method can expand the alert limits up to 150% in lateral direction and 200% in longitudinal direction, compared to those of baseline Ford model. The result is encouraging for the decimeter-level positioning requirements in autonomous vehicles.

## **2 Integrity Risk Evaluation in Civil Aviation**

Since little literature talked about the significance and importance of alert limits, even the differences between integrity and accuracy. In this section, we'll start from the integrity requirements for the navigation system and analyze the impact of alert limits on integrity risk evaluation.

### ***2.1 Integrity Requirements for Navigation System***

Both the accuracy and integrity focus the positioning errors in a certain probability. For example, as shown in Figure 1, we often define the required accuracy as the biggest position error in 95% time which corresponds to  $2\sigma$  in normal distribution. Integrity risk is a much stricter probability which is defined less than  $10^{-7}$ ~ $10^{-8}$  in most cases.

However it does not mean that integrity is a stricter accuracy in positioning results. They have obvious distinctions in function implementation.

Firstly, integrity is an index that focuses on safety-critical application. Compared to the accuracy which focuses on the best 95% test statistics (shows as the green part in the figure), the integrity risk emphasizes the impact of vehicle on hazardous situations due to the navigation system (as shown as the red part in the future). The probability of this scenario is pretty small but the impact is unacceptable for human safety.

Secondly, it is a difference between offline and online. For navigation system or sensors, accuracy is a performance index that tested and determined offline before use. Integrity is a criterion of real time online processing for particular operations. Accuracy determines whether we use this navigation system for this application. Integrity determines whether we rely on the navigation results at this epoch during this operation.

Finally, integrity includes the function of fault detection and exclusion and the ability to provide alarms when the navigation results are not reliable. Accuracy

doesn't include such functions. Last but not least, the performance of integrity also affects the performance continuity.

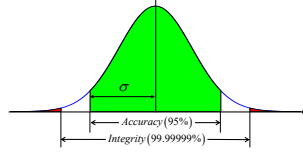


Figure 1 Relationship between accuracy and integrity

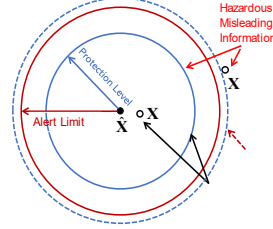


Figure 2 Relationship between PL, AL and estimated position

## 2.2 Alert Limit in Integrity Evaluation

As we mentioned in above subsection, integrity risk emphasizes the impact of vehicle on hazardous situations due to the navigation system. For specific operation, integrity risk  $PHMI$  is defined as the probability of providing a normal operation signal that is actually out of tolerance without warning the user in a given period of time. Here the maximum tolerable positioning error for an operation to safely proceed is called alert limit (AL). Correspondingly the protection level (PL) is a statistical error bound computed to guarantee the probability of error exceeding the bound is smaller than the defined integrity risk. So the integrity risk bounded by the protection level can be expressed as:

$$P\left(\left|\hat{\mathbf{X}} - \mathbf{X}\right| > AL \ \& \ PL < AL\right) \leq PHMI \quad (1)$$

where  $\mathbf{X}$  and  $\hat{\mathbf{X}}$  are the actual position and estimated position, respectively.

The relationship between PL, AL, actual and estimated position can be further shown in Figure 2. The integrity outputs can be divided into two options:

1)  $PL > AL$ . The relationship is shown as the dotted blue circle and red circle in the figure. It is easy to understand that when PL exceeds AL, the alert will be triggered immediately, no matter whether the positioning error exceeds the AL or not.

2)  $PL < AL$ . The relationship is shown as the solid blue circle and the red circle in the figure. It is an ideal circumstance and the integrity output is that the navigation position is reliable. When the integrity output is reliable but the positioning error exceeds the AL, the navigation position at this epoch is defined as the hazardous misleading information, also known as 'missed detection'.

As shown in Figure 2 and equation (1), the alert limit plays an important role in integrity evaluation. If the defined AL is too large, then the calculated PL is easy to meet the requirements, the navigation results will be evaluated as reliable in most time no matter whether the positioning result has been damaged by measurement outliers or hazardous situations. It is unacceptable in safety-critical

applications. On the contrary, if the defined AL is too small. The calculated PL is easy to exceeds the AL and trigger the alert. The navigation system will be identified as unavailable frequently due to false alarms. It doesn't reflect the real situation and is disadvantageous for the technology application. The result is that the alert limit should be objective, reasonable and reflect the navigation requirements as far as possible.

### 3 Alert Limit Determination in Autonomous Vehicles

AL requirements in civil aviation is defined with the flight operations. The alert limit is relatively simple due to the vast route before non-precision approach. Even entering the precision approach, the alert limit is still as large as tens of meters due to the wide runway. Particularly the alter limit is a constant during one certain operation. However it is no longer applicable for alert limit determination in autonomous vehicle. The next subsection will introduce the differences in detail.

#### 3.1 Scenario Difference in Autonomous Vehicles

The differences between civil aviation and autonomous vehicles are concluded in four aspects: Driving scenario, Integrity requirements, Navigation solution and sensor availability. The detailed information is shown in Table 1.

Table 1 The differences of integrity evaluation between civil aviation and autonomous vehicles.

Aspect	Item	Civil Aviation	Autonomous Vehicles
Driving Scenario	Trajectory	Smooth	Complex
	Route/Lane	~km	<4m
	Relative space	Vast	Narrow
	Weather impact	Little	Obvious
Integrity Requirements	Integrity risk unit	/h; /approach	/mile;/h
	Risk quantization	$10^{-7} \sim 10^{-8}$	To be determined (TBD)
	Alert limit range	~kilometer~10 m	TBD
	Bounding box	Simple, Cylinder	TBD
Navigation Solutions	Navigation sensors	GNSS(GPS)	GNSS/INS/LiDAR/Camera
	Navigation method	GNSS only	Multi-sensor fusion
	GNSS position model	Single point	RTK/PPP
	Aided information	SBAS/GBAS/ILS	HD map; V2X
Sensor Availability	Measurement	Pseudorange	Pseudorange/Carrier/Point Cloud...
	Positioning model	Absolute	Absolute; Relative
	Measurement performance	Similar among satellites	Diversity and complexity
	Integrity risk allocation	Equally among satellites	TBD

Based these four aspects, the integrity evaluation of autonomous vehicle is much more complex and difficult compared to that of civil aviation. The resulting integrity solution should be more rigorous due to human safety. Particularly among the above four aspects, the navigation solutions and sensor availability largely determine the accuracy and protection level computation. Then the driving scenario and integrity requirements will affect the determination of integrity risk and alert limit to a great extent. As we have mentioned in above section, besides the algorithm for positioning and protection level calculation, alert limit will finally determine the normal operation, false alarm and missed detection. The importance of alert limit is self-evident.

### 3.2 Baseline Alert Limit Determination

Actually little literature mentioned the alert limit determination in autonomous vehicles. Reference [13] is the first reviewed paper that proposed a detailed model for alert limit determination of autonomous vehicles. The input parameters include road geometry and vehicle dimension. The absolute alert limit is a trade-off in turns and the final alert limit is a relative one considering the attitude compensation. The core steps include two: Trade-off in turns and Orientation error rotation.

As shown in Figure 3, in most cases the bounding shape of AL for autonomous vehicles is defined as a cuboid box considering the vehicle dimension. The bounding box can be divided to lateral, longitudinal and vertical directions. The problem is that once the car drives into a turn, the size of the bounding box is changing due to the radius, which is shown in Figure 4. A longer longitudinal alert limit will result in a shorter latitude alert limit and vice versa. The relationship between latitude and longitudinal alert limit can be expressed in the following equation:

$$\left(\frac{y}{2}\right)^2 + \left(R - \frac{w}{2} + x\right)^2 = \left(R + \frac{w}{2}\right)^2 \quad (2)$$

where  $x$  and  $y$  are latitude and longitudinal alert limit, respectively.  $R$  and  $w$  are radius and the width of the turn. For a certain radius and width, a trade-off must be made to calculate the outputs of alert limits. There are no definite trade-off principles. The sacrifice is inevitable in one direction.

Another problem is that to meet the ideal bounding box calculated in above equation, the car needs to drive off the centreline in curved road and close to one side of the road. As shown in Figure 4, at the current epoch, the centreline of road is point  $N$ , where the center of car, also the center of the bounding box, is point  $M$ . The distance between  $M$  and  $N$  can be calculated as:

$$MN = ON - OM = \frac{w - x}{2} \quad (3)$$

where this distance is dynamic and changing due to the road type and radius. The added complexity and uncertainty to the control and navigation system make the loss outweighs the gain.

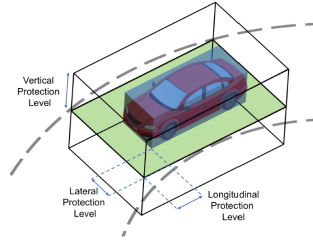


Figure 3 Bounding box definition for autonomous vehicles

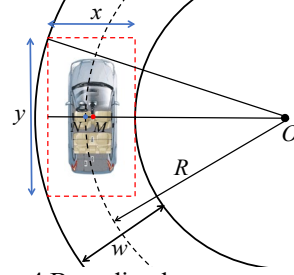


Figure 4 Bounding box geometry in a turn

It is easy to understand that it needs to modify and compensate the attitude error for a moving car positioning. However it should be noted that according to the definition of alert limit, it is an absolute bounding box under the maximum tolerable positioning error. So the attitude error should be involved in this bounding box rather than shrinking the box. In other words, attitude error is one kind of positioning error, and it has no relationship with the determination of alter limit. Furthermore, to compensate the attitude error, a lot of assumptions and compromises are made in Ford model such as: The sum of allowable longitudinal and vertical errors for freeway operation be approximately half the vehicle length; Orientation error for freeway operation is 1.5 degrees and for local streets is 0.5 degrees. These behaves conversely reduces the preciseness of the algorithm.

### ***3.3 Online Alert Limit Determination enhanced by kinematic model***

Based on the introduction of integrity and the analysis of Ford model, one important parameter not considering in Ford model is the vehicle kinematic model. As shown in Figure 5(a), when going around a curve, the direction of vehicle driving will have an apparent angle compared to the direction of head, especially in turning and roundabout. It is also another difference between civil aircraft and autonomous vehicle. Under this scenario, compared with the steering wheel and wheels, the designed cuboid box aligning with the head will not fully reflect the driving characteristic of the autonomous vehicle. Take the car head as an example, the advantage of cuboid box is to allocate the integrity risk to the left and right sides equally. However, the car has a trend to turn to the inner side due

to the kinematic model. Just shown in Figure 5(b), the outer wheels will run a bigger circle than that of the inner wheels when the car goes around a turn. The integrity risk on two sides are not balanced.

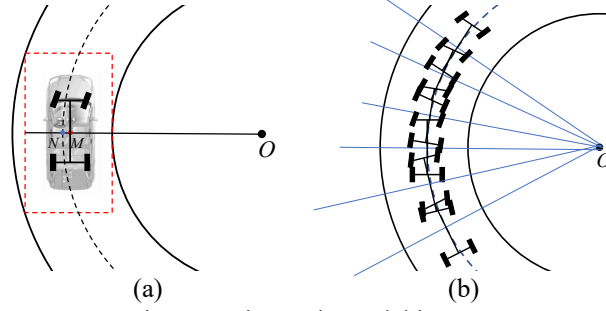


Figure 5 Kinematic model in turns

Over all, a realistic and reasonable model for integrity alert limit determination method enhanced by kinematic model is proposed. A flexible bounding box with respect to the kinematic model will replace the fixed cuboid box. In straight road, the road geometry is simple and the vehicle kinematic model is clear. A cuboid box based on the width of lane is determined using the method similar to Ford model. On the other hand, in curved road, a fan-shaped box is designed to bound the vehicle appropriately. The box is determined by the radius, the width and the design speed of the road. These parameters are easy to access from the HD map, which means the alert limit can be calculated online. With a small inner side and big outer side, the fan-shaped box is much more fit for the vehicle kinetic model. The vehicle can keep drive along the centerline of the lane to guarantee the optimum control. Starting from the fan-shaped box, it can still get the lateral and longitudinal alert limits to evaluate the localization performance. As the aim of bounding box is to avoid the vehicle itself from hazardous circumstance, the longitudinal and lateral alert limits can still be defined as the distance from the vehicle body straight to the box laterally and longitudinally, respectively. Finally, the fan-shaped bounding box is the blue shadow area in Figure 6. Focus on the positioning point  $N$ , the alert limit is shown as the red shadow area in Figure 7, where  $NE$  and  $NF$  are the lateral and longitudinal alert limit, respectively.

For the calculation of lateral and longitudinal alert limit, it is easy to find that the lateral alert limit is determined by the width of the road and the width of the vehicle:

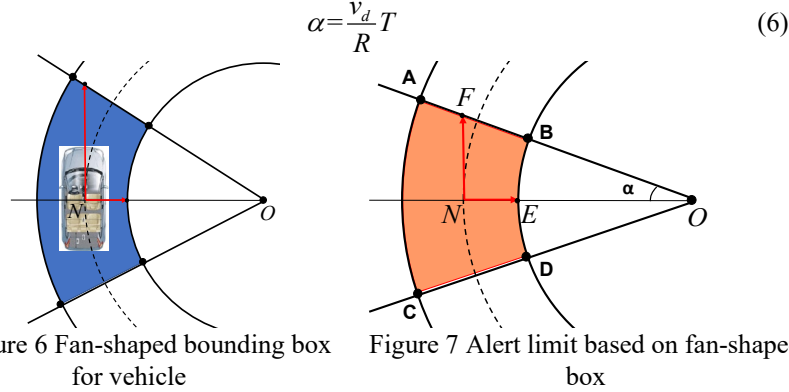
$$Lat.AL = NE = \frac{w - w_v}{2} \quad (4)$$

where  $w_v$  is the width of the vehicle.

$$Lon.AL = NF = R \arctan \alpha \quad (5)$$

where  $\alpha$  is determined by the design speed of the turn and the positioning interval time  $T$ .





Hence the shape of the bounding box is not necessary to be immutable and the alert limit can be calculated online by the road geometry, width of the vehicle directly without any trade-off and compromises.

## 4 Experiment Test and Analysis

To verify the superiority of the proposed alert limit determination method. The output comparison between Ford model and the proposed model are tested based on the American road design standard [14]. Then the alert limits based on the Chinese design specification for highway alignment are given in details [15].

### 4.1 Comparison based on American Road Design Standard

Table 2 and Table 3 show the alert limits based on America standard road types and vehicle types. Particular the road widths used in freeway operation and local road are 3.6m and 3.0m, respectively. The radiuses of turns are 150m and 20m, respectively.

Table 2 Alert limits for America freeway operation

Vehicle Type	Ford Model		Proposed Model	
	Lat.AL/m	Lon.AL/m	Lat.AL/m	Lon.AL/m
Mid-Size	0.72	1.40	0.86	2.78
Full-Size	0.66	1.40	0.83	2.78
Standard Pickup	0.62	1.40	0.80	2.78
Passenger Vehicle Limits	0.57	1.40	0.75	2.78
6-Wheel Pickup	0.40	1.40	0.59	2.78

As the tables shown, compared to those calculated by Ford model, the lateral alert limits determined by the proposed model are broadened from 20%~50%. The longitudinal alert limits are broadened about two times. The safe spaces between

the vehicle and the lane are maximized to the full. It is significant for the autonomous vehicle navigation with relatively less stringent alert limits.

Table 3 Alert limits for America local road

Vehicle Type	Ford Model		Proposed Model	
	Lat.AL/m	Lon.AL/m	Lat.AL/m	Lon.AL/m
Mid-Size	0.44	0.44	0.58	0.83
Full-Size	0.38	0.38	0.53	0.83
Standard Pickup	0.34	0.34	0.50	0.83
Passenger Vehicle Limits	0.29	0.29	0.45	0.83

#### 4.2 Alert limit based on Chinese Design Specification for Highway Alignment

According to the Chinese design specification for highway alignment, the road can be divided into five grades with different design speed and road width. The vehicles can be divided into five types according to the vehicle size. Table 4 gives the detailed lateral and longitudinal alert limits based on road grade and vehicle type.

Table 4 Alert limits based on Chinese design specification for highway alignment

Road grade Vehicle Type	Freeway	first-class highway	second-class highway	third-class highway	forth-class highway
passenger car	0.98m/2.78m	0.98m/2.22m	0.85m/1.67m	0.85m/1.11m	0.73m/0.83m
passenger bus	0.60m/2.78m	0.6m/2.22m	0.48m/1.67m	0.48m/1.11m	0.35m/0.83m
articulated bus	0.63m/2.78m	0.63m/2.22m	0.50m/1.67m	0.50m/1.11m	0.38m/0.83m
Truck	0.63m/2.78m	0.63m/2.22m	0.50m/1.67m	0.50m/1.11m	0.38m/0.83m
articulated vehicle	0.60m/2.78m	0.6m/2.22m	0.48m/1.67m	0.48m/1.11m	0.35m/0.83m

## 5 Conclusion

Based on the analysis of the importance of integrity and the application differences from civil aviation to autonomous vehicles, an improved alert limit determination method enhanced by the kinematic model is proposed in this paper. The integrity risk on two sides are allocated respect to the road geometry and kinematic model. A fan-shaped bounding box is more reasonable to cover the safe-critical areas. The experiment test results compared with those of the Ford model also verified the superiority of the proposed method. The alert limits

calculated based on the Chinese standards can give some references for the navigation integrity for autonomous vehicles.

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